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Influence of substrate microrelief on the Fréedericksz transition in a thin nematic cell

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An effect of substrate surface regular wavy microrelief on the Freedericksz transition in a thin homeotropically aligned nematic cell near the smectic-*A*–nematic second-order transition temperature is theoretically investigated. It is shown that because of the suppression of an interfacial smectic-*A* structure, the Fréedericksz critical field for the cell with substrates having a sufficiently sharp microrelief should be significantly lower than that for the cell of the same thickness, but with perfectly flat substrates. [S1063-651X(99)03805-2]

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I. INTRODUCTION

It is known that an interaction between liquid crystal (LC) molecules and the boundary surfaces, such as a free surface and a solid substrate, not only gives rise to changes in orientational order in the interfacial LC region, but also induces interfacial smectic order [1,2]. At present, the existence of the surface-induced smectic structures in both isotropic and nematic LC phases is well established, and these interfacial structures are the objects of intensive experimental [3–7] and theoretical [8–11] investigation.

One of the most important problems that arises in such investigations, is the influence of a solid substrate surface microrelief (roughness) on the interfacial smectic structures. In fact, only the free surface of the LC can be considered as a perfectly flat boundary one. In most experimentally realizable situations, the solid substrate surface always possesses a certain natural microrelief. In addition, the substrate surface microrelief of various shapes can be artificially created, for example, by evaporation of thin films of some oxides (SiO, GeO, etc.) [12], or photolitographic techniques [13–15]. In Refs. [16,17] it has been shown that the regular substrate surface microrelief deforms adjacent smectic layers and hinders a formation of the interfacial smectic structure. According to the estimate performed in these papers, the regular wavy microrelief of period $d\sim0.5\ \mu$m and amplitude $U_0\sim0.02\ \mu$m can suppress almost completely the substrate surface-induced smectic-*A* (Sm*A*) structure.

One can then ask: how can one check the validity of this theoretical result? The most detailed information of the interfacial smectic structure in LC can be obtained from experiments using small-angle x-ray scattering from the LC interface [3–5]. However, in most such experiments only the free surface of LC has been studied, because an x-ray study of the LC–solid wall interface is very complicated. Nevertheless, in Ref. [18] it has been shown that the existence or absence of the interfacial Sm*A* structure can be revealed indirectly by the well-known Fréedericksz transition [19,20] in a very thin homeotropically aligned nematic cell of thickness $h\sim2-3\ \mu$m. When the interfacial smectic structure is formed near the substrates of such a cell, a transverse external magnetic field $\vec{H}$ would be unable to deform this surface-induced smectic region. Then, if the interfacial smectic order penetrates into the nematic bulk for a distance $\sim\xi_P$, the effective thickness h^* of the deformed nematic layer in the cell would be reduced by order $2\xi_P$ ($h^*\sim h-2\xi_P$). Hence, the Freedericksz transition critical field H_c, which is directly proportional to $1/h^*$, would substantially increase due to the existence of an interfacial Sm*A* structure in the thin nematic cell. In fact, an anomalous increase in the critical field H_c has been observed in an ultrathin ($h\approx2.6\ \mu$m), homeotropically aligned octylcyanobiphenyl (8 CB) cell near T_{NA}, the nematic–smectic-*A* (*N*–Sm*A*) second-order transition temperature, and such behavior can be ascribed to smectic layering near the substrates [18].

This result suggests how one can check the validity of the theoretical prediction [16,17] of the suppression of the interfacial Sm*A* structure in the nematic LC (NLC) by a regular microrelief on the solid substrate surface. It is necessary to measure the Fréedericksz critical field in the vicinity of the second-order N–Sm*A* phase transition in two homeotropically aligned nematic cells of the same thickness $h\sim2-3\ \mu$m. The first cell should have substrates with perfectly flat surfaces, and in the second one the substrate surfaces should possess a sufficiently sharp regular microrelief. If the Freedericksz critical field for the first cell is higher than that for the second one, the validity of the above theoretical results could be confirmed. One can, however, ask: How sharp should the regular microrelief be (i.e., how large, or small, should be the period d and the amplitude U_0) to result in a difference between the critical fields for the two above cells to exceed achievable (~1% [18]) measurement accuracy? How close must we approach the bulk N–Sm*A* transition temperature to detect the effect of the interfacial smectic-*A* structure on the Fréedericksz transition critical field? In order to answer these questions, we have to investigate theoretically the influence of the smectic-*A* structure, which is induced by the solid substrate surface with regular microrelief in NLC, on the Fréedericksz transition. Such a theoretical investigation is an aim of the present paper.

We start in the next section with a description of the smectic-*A* phase induced by the substrate surface with regular wavy microrelief in NLC. This description is a modified version of the oversimplified theory proposed in Refs. [16,17]. In Sec. III the results obtained in Sec. II are used in a calculation of the Fréedericksz critical field for a thin nematic cell with due regard for the deformed surface-induced Sm*A* structure. Section IV presents the results of numerical calculation of the critical field followed by a discussion.

II. DESCRIPTION OF THE Sm*A* PHASE INDUCED BY A DEFORMED SURFACE IN NLC

Let us consider the nematic layer in contact with the solid substrate surface having a wavy microrelief described by a one-dimensional harmonic function, for example,

$$U_0(x)=U_0\cos[(2\pi/d)x], \tag{1}$$

where U_0 and d are the amplitude and period of the surface microrelief, respectively. The NLC layer is assumed to be homeotropically aligned (in the bulk sample the director $\vec{n}$ is parallel to the z axis, normal to the substrate surface). Let us also assume that our LC sample is in the vicinity of the second order N-Sm*A* phase transition (the NLC temperature is slightly above the transition point) and the orientational order is perfect (the orientational order parameter $S=1$, i.e., the long axes of all molecules are parallel to the director $\vec{n}$). The latter assumption is reasonable enough because most LC's undergo a second order N-Sm*A* phase transition sufficiently far from the clearing point and the actual orientational order is very close to perfect.

Let us assume that due to the interaction between mesogenic molecules and the substrate surface the latter induces a positionally ordered Sm*A* structure with a period equal to the molecular length l. If the molecules within the first interfacial smectic layer are assumed to be rigidly anchored to the substrate surface, then this layer should be distorted by the substrate sinusoidal relief, and this distortion due to the small smectic layer compressibility can be transmitted to the neighboring layers. It is clear that the deformation of the interfacial smectic structure should affect its translational order, i.e., the value of the smectic order parameter. On the other hand, the depth of penetration of the surface microrelief-induced deformation depends on the smectic layer compressibility, which is determined in turn by the translational order of the interfacial smectic structure. Thus, to describe completely the LC region near the substrate surface with a microrelief we must determine simultaneously the interfacial smectic order-parameter profile and the penetration of the surface microrelief-induced deformation into the sample bulk.

In order to solve this problem we must know the expression for the free energy density in the interfacial LC layer. This expression should contain two contributions. The first contribution is the free energy density of the deformed nematic liquid crystal [19,20]

$$f_N=(K_1/2)(\operatorname{div}\vec{n})^2+(K_2/2)(\vec{n}\cdot\operatorname{rot}\vec{n})^2+(K_3/2)(\vec{n}\times\operatorname{rot}\vec{n})^2, \tag{2}$$

where $K_{1,2,3}$ are the Frank splay, twist, and bend elastic constants, respectively. If the substrate surface wavy deformation is assumed to be weak enough $[(2\pi/d)U_0\ll 1]$, then the director components are related to the one-dimensional wavy deformation $U(x,z)$ by the equations

$$n_x\approx-\partial U/\partial x,\quad n_y=0,\quad n_z\approx 1. \tag{3}$$

Substituting these relations into Eq. (2) one obtains

$$f_N\approx(K_1/2)(\partial^2U/\partial x^2)^2+(K_3/2)(\partial^2U/\partial x\partial z)^2. \tag{4}$$

It should be noticed that since the temperature of the LC under consideration is in the vicinity of the second order N−Sm*A* phase transition, the bend elastic constant K_3 in Eq. (4) is not similar to that in the pure nematic phase. Near the second-order N−Sm*A* transition point the smectic short-order fluctuations occur in the nematic bulk phase. These smectic order fluctuations are not induced by the substrate surface and must be considered separately from the surface-induced smectic structure. As it will be seen below, they give rise to a renormalization of the elastic constant K_3.

The second contribution is the free energy density of the surface-induced Sm*A* phase. If $\sigma(z)$ is the smectic order parameter and $U(x,z)$ is the shift of smectic layers due to the substrate surface microrelief, then the surface-induced smectic-*A* phase in the vicinity of the second-order N−Sm*A* phase transition is described by the density wave

$$\rho(x,z)=\rho_0\{1+\sigma(z)\cos[2\pi(z-U(x,z))/l]\}, \tag{5}$$

where ρ_0 is the average density of the liquid crystal molecules, and the free-energy density of the Sm*A* phase is given by the Landau−de Gennes expression:

$$f_{\mathrm{Sm}A}=(A/2)\sigma^2+(C/4)\sigma^4+(L/2)(d\sigma/dz)^2+(B/2)(\partial U/\partial z)^2, \tag{6}$$

where $A=\alpha(T-T_{NA})$, α and C are the temperature-independent constants, T is the temperature of the system, T_{NA} is the N−Sm *A* phase transition temperature, L is the elastic constant which, in the framework of a mean-field theory, determines the longitudinal correlation length ξ for smectic fluctuations in the vicinity of the second-order N−Sm*A* phase transition as [19,20]

$$\xi=(L/A)^{1/2}, \tag{7}$$

and B is the smectic layer compressibility. It should be noted that the two gradient terms in Eq. (6) are due to the coordinate dependence of the absolute value of the smectic order parameter σ and the smectic layer elastic deformation, respectively. Further, according to Ref. [20], the smectic layer compressibility B can be represented as

$$B=B_0\sigma^2, \tag{8}$$

and the elastic constant B_0 is related to the constant L by the equation

$$B_0=L(2\pi/l)^2. \tag{9}$$

Adding expressions (4) and (6) and taking into account Eq. (8) one can obtain the following equation for the total free energy density of the LC layer near the deformed substrate surface

$$f=f_N+f_{\mathrm{Sm}A}=(K_1/2)(\partial^2U/\partial x^2)^2+(K_3/2)(\partial^2U/\partial x\,\partial z)^2+(A/2)\sigma^2+(C/4)\sigma^4+(L/2)(d\sigma/dz)^2+(B_0/2)\sigma^2(\partial U/\partial z)^2. \tag{10}$$

In order to determine the total free energy of the interfacial region we must integrate the free energy density (10) over the space above the substrate surface and add to this result the energy of direct interaction between the liquid crystal molecules and the substrate. In previous papers on the surface-induced smectic-*A* phase [8,9] this interaction was simulated by a short-range orienting field which acts directly only on the molecules in contact with the boundary surface. The energy of such an interaction can be written as

$$G(z,\vartheta)=-G_0(3/2\cos^2\vartheta-1/2)\,\delta(z-U_0(x)),$$

where ϑ is the angle between long axes of the LC molecules and the normal to the boundary surface, G_0 is the interaction constant, and $\delta(z-U_0(x))$ is a well-known Dirac function. Since the orientational order in the system under consideration is assumed to be perfect ($\cos\vartheta\rightarrow1$) this potential can be represented as

$$G(z)=-G_0\,\delta(z-U_0(x)), \tag{11}$$

and the energy of interaction per unit substrate square is equal to

$$F_s=\int_0^\infty G(z)\rho(x,z)\,dz=-G_0\rho_0-G_0\rho_0\sigma_0, \tag{12}$$

where σ_0 is the value of the smectic order parameter at the substrate surface. Finally, the total free energy of the LC interfacial layer per unit substrate square can be represented as

$$F=\int_0^\infty\overline{f}(z)\,dz-G_0\rho_0-G_0\rho_0\sigma_0, \tag{13}$$

where $\overline{f}(z)$ is the free energy density of the LC interfacial layer averaged over the XY plane. If we take the expression for $U(x,z)$ in the form

$$U(x,z)=U(z)\cos[(2\pi/d)x],$$

$$U(x,z)|_{z=0}=U_0\cos[(2\pi/d)x], \tag{14}$$

then the expression for $\overline{f}(z)$ is the following:

$$\overline{f}=(A/2)\sigma^2+(C/4)\sigma^4+(L/2)(d\sigma/dz)^2+(B_0/4)\sigma^2(dU/dz)^2+(K_1/4)(2\pi/d)^4U^2+(K_3/4)(2\pi/d)^2(dU/dz)^2. \tag{15}$$

Substituting Eq. (15) into expression (13) for the total free energy of the interfacial layer and minimizing the latter with respect to $\sigma(z)$, $U(z)$, and σ_0, we obtain the following Euler-Lagrange equations:

$$L(d^2\sigma/dz^2)-A\sigma-C\sigma^3-(B_0/2)\sigma(dU/dz)^2=0, \tag{16}$$

$$\frac{d}{dz}[(B_0/2)\sigma^2(dU/dz)]+(K_3/2)(2\pi/d)^2(d^2U/dz^2)-(K_1/2)(2\pi/d)^4U=0, \tag{17}$$

and the boundary condition

$$\left.\frac{\partial\overline{f}}{\partial(\partial\sigma/\partial z)}\right|_{z=0}=-G_0\rho_0. \tag{18}$$

A solution of Eqs. (16), (17) with boundary condition (18), as well as under the condition of complete decay of both the interfacial smectic structure and the substrate surface microrelief-induced deformation with penetration into the nematic bulk ($\sigma\rightarrow0$, $d\sigma/dz\rightarrow0$, $U\rightarrow0$, $dU/dz\rightarrow0$ at $z\rightarrow\infty$), determines the interfacial smectic order parameter profile and the deformation. Because of their nonlinearity, however, these equations can be solved only numerically. Nevertheless, let us attempt to obtain the approximative analytical solution. We can use the following approach. Even for a perfectly flat substrate, the surface-induced smectic order should decay with penetration into the nematic bulk at a distance of order of the longitudinal correlation length ξ for smectic fluctuations [21]. According to experimental data [22], at a temperature about 0.1 K higher above the second-order N–Sm*A* phase transition, this correlation length is of order 0.1 μm. On the other hand, if the period of the substrate surface wavy relief d is of order of 1 μm, then the depth of the penetration of the surface-induced deformation into the nematic bulk should be of the same order [19]. Therefore it is reasonable to assume that $\sigma(z)$ must decay rapidly in comparison with $U(z)$, or $\sigma(z)$ is a rapidly varying function and $U(z)$ is a slowly varying one. Then in the third term of Eq. (17) $U(z)$ can be approximately considered as a constant value, and one can set $U(z)\approx U_0$. The subsequent integration of this equation over any interval z_2-z_1 within a region of the existence of the interfacial Sm*A* phase yields

$$[B_0\sigma^2(z_1)+K_3(2\pi/d)^2](dU/dz_1)\approx[B_0\sigma^2(z_2)+K_3(2\pi/d)^2](dU/dz_2)-K_1(2\pi/d)^4U_0(z_2-z_1). \tag{19}$$

If at the point z_2 the interfacial smectic order decayed sufficiently [$\sigma(z_2)\approx0$], then dU/dz_2 can be determined from the equation

$$dU/dz_2=-(K_1/K_3)^{1/2}(2\pi/d)U, \tag{20}$$

which describes the decay of a wavy deformation in the homeotropically aligned NLC [19]. Hence,

$$[B_0\sigma^2(z_1)+K_3(2\pi/d)^2](dU/dz_1) \approx -(K_1K_3)^{1/2}(2\pi/d)^3U_0-K_1(2\pi/d)^4U_0(z_2-z_1), \tag{21}$$

and the ratio of the second term on the right-hand side of Eq. (21) to the first one is equal to $(K_1/K_3)^{1/2}[2\pi(z_2-z_1)/d]$. Since the depth of penetration of the interfacial smectic order into the nematic bulk should be an order of magnitude smaller than the period d, and near the second-order N–Sm*A* phase transition, the bend elastic constant K_3 is significantly larger than the splay elastic constant K_1 [19,20], and the value of this ratio should be much smaller than unity. In this situation the second term on the right-hand side of Eq. (21) can be omitted. This yields

$$(dU/dz)\approx -\frac{(K_1K_3)^{1/2}(2\pi/d)^3U_0}{[B_0\sigma^2(z)+K_3(2\pi/d)^2]}. \tag{22}$$

Substituting this approximate expression into Eq. (15), one can integrate it once and obtain the solution

$$(d\sigma/dz)\approx -\sqrt{W(\sigma,U_0,d)}, \tag{23}$$

where

$$W(\sigma,U_0,d)=\xi^{-2}\sigma^2+C^*\sigma^4+(K_1/2L)(2\pi/d)^4U_0^2 \times\{1-[(B_0/K_3)(d/2\pi)^2\sigma^2+1]^{-1}\}, \tag{24}$$

and $C^*=C/2L$. From Eq. (23) one can deduce directly the equation

$$z=\int_{\sigma(z)}^{\sigma_0}\frac{d\sigma}{\sqrt{W(\sigma,U_0,d)}}, \tag{25}$$

which determines the surface-induced smectic order-parameter profile $\sigma(z)$. Combining Eq. (22) with the boundary condition (18) and taking into account that

$$\left.\frac{\partial \overline{f}}{\partial(\partial\sigma/\partial z)}\right|_{z=0}=L\left.\frac{d\sigma}{dz}\right|_{z=0}, \tag{26}$$

one can easily derive the algebraic equation

$$W(\sigma_0,U_0,d)=(G_0\rho_0/L)^2, \tag{27}$$

which determines the value σ_0 of the smectic order parameter at the substrate surface.

The relations obtained above allow us to determine the smectic order-parameter profile that describes the interfacial Sm*A* structure near the solid substrate surface with a regular wavy microrelief in the vicinity of the second-order N–Sm*A* phase transition. Such profiles, obtained for different values of the amplitude U_0 and fixed period $d=1$ μm of the microrelief, are shown in Fig. 1. The numerical calculation has been performed with the following values of parameters: $(T-T_{NA})/T_{NA}=10^{-4}$; $T_{NA}\approx 307$ K; $\xi=0.16$ μm; $K_1=10^{-6}$ dyn; $l=3\times10^{-7}$ cm (data for LC 8CB from Ref. [22]); $C^*\approx 1200$ [23]. The magnitude of B_0 has been determined from the relationship $B_0/K_1\approx 1/l^2$ [19,20], and the ratio K_3/K_1 has been taken as

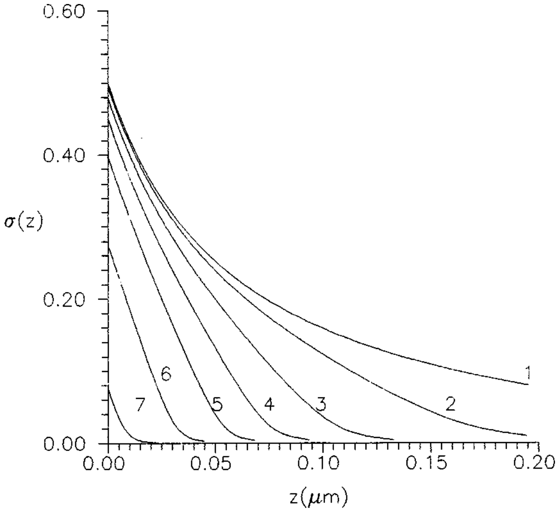

FIG. 1. The interfacial smectic order-parameter profiles for various amplitudes of the substrate surface microrelief. $d=1$ μm. 1, $U_0=0$; 2, $U_0=0.01$ μm; 3, $U_0=0.02$ μm; 4, $U_0=0.03$ μm; 5, $U_0=0.04$ μm; 6, $U_0=0.05$ μm; 7, $U_0=0.06$ μm.

$$K_3/K_1\approx 1+(k_BT/6)(\pi\xi/l^2K_1), \tag{28}$$

where k_B is the Bolzmann constant. This equation can be obtained by using the results of Ref. [24], which deals with the effect of the short range smectic order fluctuations in the bulk nematic phase in the vicinity of the second-order N–Sm*A* phase transition on the bend elastic constant K_3. As for value of the parameter $G_0\rho_0/L$, its choice must be somewhat arbitrary because we have no detailed information on the direct interaction between the LC molecules and the substrate surface. Therefore, this parameter has been chosen to

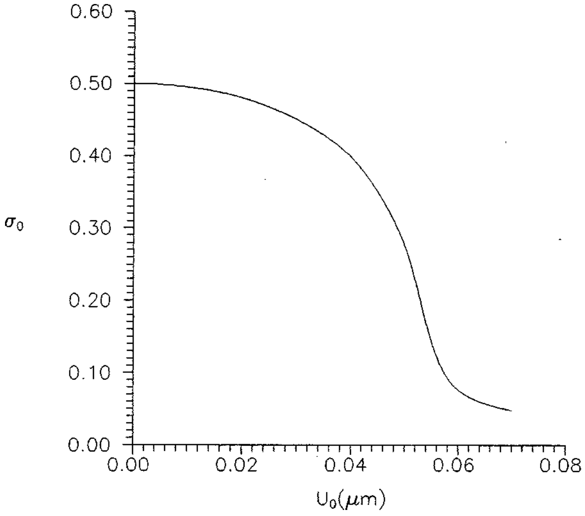

FIG. 2. Dependence of the smectic order parameter at the substrate surface σ_0 on the amplitude U_0 of the wavy microrelief. $d=1$ μm.

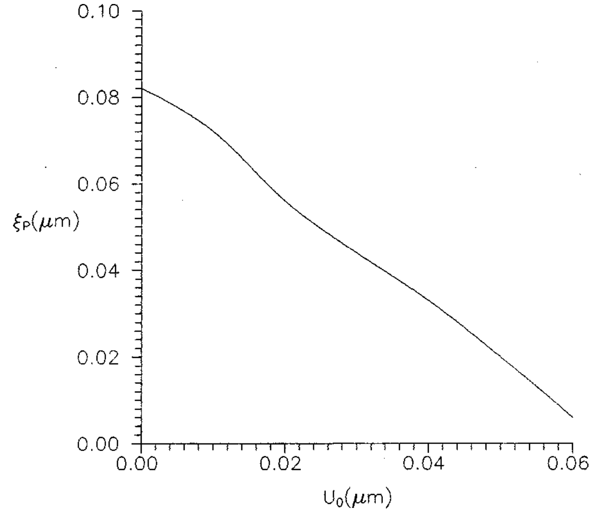

FIG. 3. Dependence of the depth of penetration of the surface-induced smectic order into the nematic bulk on the amplitude of the wavy microrelief. $d=1$ μm; $(T-T_{NA})/T_{NA}=10^{-4}$.

provide a reasonable value, for example $\sigma_0=0.5$, of the smectic order parameter at the perfectly flat substrate surface $(U_0=0)$. From Fig. 1 it is clearly seen that the substrate surface deformation gives rise to the suppression of the interfacial smectic-*A* structure, and the sharper the substrate surface microrelief (the larger the amplitude U_0 at the fixed period d), the ''weaker'' the interfacial smectic order. Both the smectic order parameter σ_0 at the substrate surface (see Fig. 2) and the penetration depth ξ_P of the smectic order into the nematic bulk [ξ_P is the distance from the substrate at which the smectic order parameter $\sigma(z)$ is e times smaller than σ_0] (see Fig. 3) decrease with increasing microrelief amplitude U_0. From Fig. 3 it is also seen that the depth ξ_P decays almost linearly with U_0.

III. CALCULATION OF FRÉEDERICKSZ TRANSITION CRITICAL FIELD FOR NLC CELL WITH DUE REGARD FOR SmA PHASE INDUCED BY THE SUBSTRATE SURFACE WITH REGULAR MICRORELIEF

Let us consider a thin homeotropically aligned NLC cell which is placed into a homogeneous magnetic field $\vec{H}$ perpendicular to the nematic director $\vec{n}$. When the magnetic field achieves a certain critical value H_c, the director $\vec{n}$ reorients in what is commonly known as a Fréedericksz transition [19,20]. In a ''pure'' homogeneous NLC cell, i.e., without the interfacial smectic layering, the critical field is equal to

$$H_c^*=\frac{\pi}{h}\left(\frac{K_3}{\chi_a}\right)^{1/2}, \tag{29}$$

where χ_a is the LC magnetic susceptibility anisotropy. As discussed in the Introduction, when interfacial smectic layering is induced by the substrate surface in the NLC cell, an additional hindrance to the director reorientation appears that results, in turn, in an increase of the critical field H_c.

A simple way to take into account the effect of the substrate surface-induced smectic-*A* structure on the critical field H_c has been offered in Ref. [18]. Since a tilt of the director $\vec{n}$ by a small angle ϑ relative to its initial homeotropic orientation is accompanied by deformation of the interfacial smectic layering, the well-known expression for the elastic deformation energy density of the homeotropically aligned NLC in a transverse magnetic field $\vec{H}$ [19,20] should be supplemented by an interfacial smectic layer deformation energy $D(z)\vartheta^2$, where $D(z)$ is a parameter which, like the smectic layer compressibility *B*, is proportional to σ^2 [18]. This parameter plays the role of an ''additional stabilizing field,'' which has a maximum strength near the substrates of the cell. Then, taking into account the symmetry of the cell with respect to its center $(z=h/2)$, the total elastic deformation energy of the sample can be written as [18]

$$F_d=\int_0^{h/2}[K_3(d\vartheta/dz)^2-\chi_a H^2\vartheta^2+D(z)\vartheta^2]\,dz, \tag{30}$$

where K_3 and χ_a are assumed to be spatially uniform.

In order to determine the Fréedericksz critical field H_c as a function of the cell thickness h and the temperature T, we must choose a suitable dependence for $\vartheta(z)$ which is (1) symmetrical with respect to the center of the cell, and (2) satisfies the boundary condition of rigid LC-substrate anchoring, namely,

$$\vartheta(z=0)=\vartheta(z=h)=0. \tag{31}$$

Then one can expand $\vartheta(z)$ in a Fourier series

$$\vartheta(z)=\sum_{\nu=1}^{\infty}\vartheta_\nu\sin\left(\frac{\pi\nu}{h}z\right), \tag{32}$$

and, as in Ref. [19], retain only the first term of this expansion, i.e.,

$$\vartheta(z)\approx\vartheta_0\sin\left(\frac{\pi}{h}z\right). \tag{33}$$

Inserting this expression into Eq. (30), we obtain the nematic deformation energy per unit square of the substrate

$$F_d\approx\frac{\vartheta_0^2 h}{4}\left[\left(K_3\frac{\pi^2}{h^2}-\chi_a H^2\right)+(4/h)\int_0^{h/2}D(z)\sin^2\left(\frac{\pi}{h}z\right)dz\right]. \tag{34}$$

As mentioned above, the parameter $D(z)$ should be proportional to σ^2, and, hence, it can be written as $D_0\sigma^2(z)$, where D_0 is a certain elastic constant which determines the hardness of the smectic layers to the tilt of the director $\vec{n}$ relative to the layer normal.

When the magnetic field H is lower than the critical value H_c, the nondistorted state of the sample is stable, and the deformation energy F_d must be positive for any $\vartheta_0\neq 0$. Then the critical field H_c can be determined from a condition $F_d=0$, which yields

$$H_c^2 = \frac{K_3\pi^2}{\chi_a h^2}\left[1 + 4\frac{D_0 h}{K_3\pi^2}\int_0^{h/2}\sigma^2(z)\sin^2\left(\frac{\pi}{h}z\right)dz\right]. \quad (35)$$

The ratio of H_c to the Fréedericksz critical field H_c^* for a "pure" and homogeneous nematic sample, i.e., without the interfacial smectic structure, is equal to

$$H_c/H_c^* = \sqrt{1 + 4\frac{D_0 h}{K_3\pi^2}\int_0^{h/2}\sigma^2(z)\sin^2\left(\frac{\pi}{h}z\right)dz}. \quad (36)$$

If we remember that the surface-induced smectic order almost completely decays at a distance from the substrate of the order of $\sim\xi_P\sim 0.1\mu$, and the thicknesss of even "ultranarrow" cells used in [18] was $h\sim 2-3\ \mu$m, then in the integral in the right-hand side of Eq. (36) $\sin^2[(\pi/h)z]$ can be approximately written as $\approx(\pi^2/h^2)z^2$, and the ratio H_c/H_c^* can be represented as

$$H_c/H_c^* \approx \sqrt{1 + 4\frac{D_0}{K_3 h}\int_0^{\infty}\sigma^2(z)z^2\,dz}. \quad (37)$$

From Eq. (37) it is clearly seen that the influence of the interfacial smectic structure on the Fréedericksz critical field can be significant only for sufficiently thin cells, because, for thick cells ($h\rightarrow\infty$), the right-hand side of this equation goes to unity, i.e., the critical field for a "thick" NLC sample with interfacial smectic layers is indistinguishable from that for a "pure" nematic sample of the same thickness. Finally, if the integral term on the right-hand side of Eq. (37) is assumed to be much less than unity, then the ratio H_c/H_c^* can be approximated as

$$H_c/H_c^* \approx 1 + 2\frac{D_0}{K_3 h}\int_0^{\infty}\sigma^2(z)z^2\,dz. \quad (38)$$

By analogy with Ref. [18], one can also define the quantity $\Omega = H_c/H_c^* - 1$:

$$\Omega \approx 2\frac{D_0}{K_3 h}\int_0^{\infty}\sigma^2(z)z^2\,dz. \quad (39)$$

IV. RESULTS OF NUMERICAL CALCULATION AND DISCUSSION

As seen from Eq. (39), to calculate the temperature dependence of the parameter Ω, we must know, in addition to the smectic order-parameter profile $\sigma(z)$ and the temperature dependence of the elastic constant K_3, the magnitude of the parameter D_0. This magnitude we can determine as follows. Taking the cell thickness, as in Ref. [18], to be equal to $h\approx 2.5\ \mu$m, and assuming the cell substrates to be perfectly flat ($U_0=0$), we determine the parameter D_0 such that we obtain the value Ω at certain temperature T, say 0.2 K higher than T_{NA}, similar to that found experimentally. From Fig. 1 in Ref. [18] one can find $\Omega(T=T_{NA}+0.2\ \mathrm{K})\approx 0.13$. For this value of Ω we find $D_0=5.5\times 10^6$ erg/cm^3. Such a value of D_0 can be used in a numerical calculation of Ω at any temperature T for both cells with perfectly flat substrates and that with substrates having the wavy microrelief. The temperature dependence of the parameter Ω calculated in such a manner for the cell with perfectly flat substrates ($h=2.5\ \mu$m) is shown in Fig. 4. For comparison, the experimental data from Ref. [18] are also brought here. It is easily seen that the results of the calculation are in a satisfactory agreement with the experimental ones.

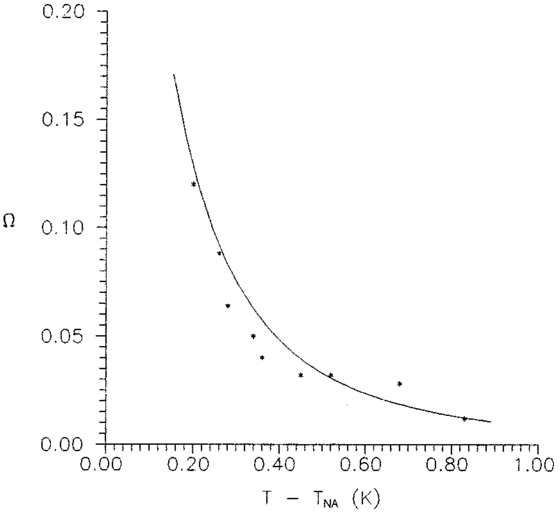

FIG. 4. Temperature dependence of the parameter Ω for the cell with perfectly flat substrates. $h=2.5\ \mu$m; $D_0=5.5\times 10^6$ erg/cm^3. (*) indicate experimental points from Ref. [18].

Analogous dependences calculated for different values of the amplitude U_0 of the wavy microrelief on the substrate surface ($d=1\ \mu$m) are shown in Fig. 5. From this figure one can conclude that the effect of the interfacial Sm*A* structure on magnitude of the Fréedericksz critical field can be detected if the temperature of the system under investigation is lower than $\sim T_{NA}+1$ K. In the opposite case, the parameter Ω is smaller than the actually achievable ($\sim$1% [18]) accuracy of a measurement of the critical field. Figure 5 also

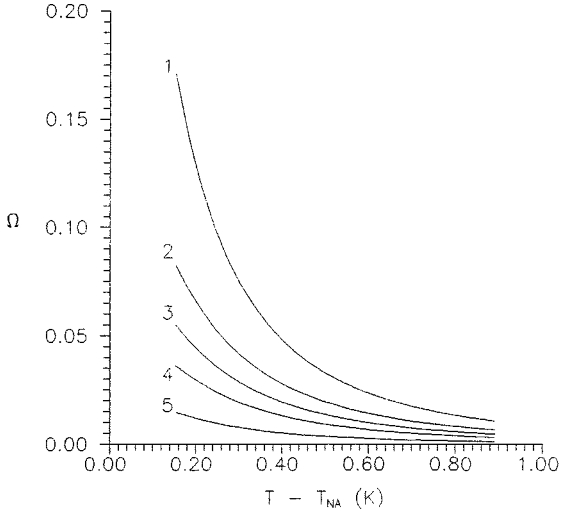

FIG. 5. Temperature dependences of the parameter Ω for different values of the amplitude of the wavy microrelief. $d=1\mu$m. 1, $U_0=0$; 2, $U_0=0.01\ \mu$m; 3, $U_0=0.015\ \mu$m; 4, $U_0=0.02\ \mu$m; 5, $U_0=0.03\ \mu$m.

demonstrates a strong decay of Ω with increasing U_0, and the closer the sample to the bulk N–Sm*A* transition point, the larger the difference in Ω for the cell with flat substrates and for that with substrates having the microrelief. For example, for $T=T_{NA}+0.2$ K, the parameter Ω for the cell with flat substrates is equal to 0.13, whereas for $U_0=0.01$ μm, $\Omega=0.07$; for $U_0=0.015$ μm, $\Omega=0.045$; for $U_0=0.02$ μm, $\Omega=0.03$; and finally, for $U_0\geqslant 0.03$ μm, $\Omega<0.01$. Consequently, in the vicinity of the bulk N–Sm*A* phase transition temperature, the critical field H_c for the thin cell with substrates having a sufficiently sharp wavy microrelief should be similar to that for the Freédericksz transition in a ''pure'' and homogeneous nematic sample of the same thickness *h*.

Thus, by observing the Freédericksz transition near the bulk N– Sm*A* transition temperature ($T\sim T_{NA}+0.2$ K) in two homeotropically oriented 8 CB cells of the same thickness $h\sim 2.5$ μm, one of which has perfectly flat substrates, and another has substrates with regular wavy microrelief ($U_0\sim 0.03$ μm, $d\sim 1$ μm), we should find the critical field for the first cell to be about 13% higher than that for the second one. This difference in magnitude of H_c is much larger than the achievable experimental accuracy of the Freédericksz critical field, and its detection could be considered as a confirmation of the theory [16,17] which predicts suppression of the interfacial smectic-*A* (Sm*A*) structure by the regular microrelief on the solid substrate surface.

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